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Proof of the Double Bubble Conjecture in $\mathbf{R}^n$

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Abstract The least-area hypersurface enclosing and separating two given volumes in $\mathbf{R}^n$ is the standard double bubble.

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1 Introduction

1.1 The Double Bubble Conjecture

We extend the proof of the double bubble theorem [11] from $\mathbf{R}^3$ to $\mathbf{R}^n$.

Theorem 1.1 (Double Bubble Conjecture) *The least-area hypersurface enclosing and separating two given volumes in* $\mathbf{R}^n$ *is the standard double soap bubble of Fig.* 1, *consisting of three* $(n-1)$*-dimensional spherical caps intersecting at* 120° *angles.* (*For the case of equal volumes, the middle cap is a flat disk.*)

In 1990, Foisy et al. [4] proved the Double Bubble Conjecture in $\mathbf{R}^2$. In 1995, Hass et al. [5, 6] used a computer to prove the conjecture for the case of equal volumes in $\mathbf{R}^3$.

Arguments since have relied on the Hutchings structure theorem (Theorem 3.1), stating roughly that the only possible nonstandard minimal double bubbles are rotationally symmetric about an axis and consist of "trees" of annular bands wrapped

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